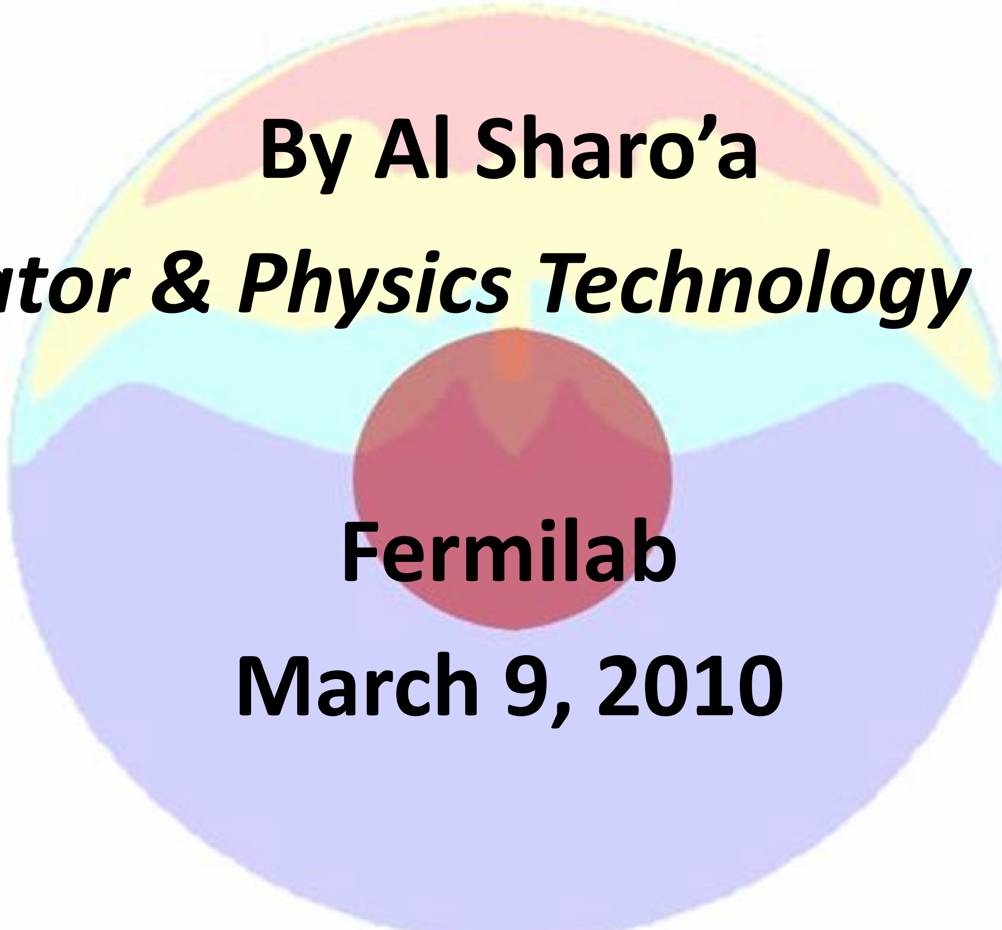
Analysis of the Transient Natural Convection Driven by Energy Deposition inside High-Pressure RF Cavities



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Accelerator & Physics Technology Seminar

Fermilab

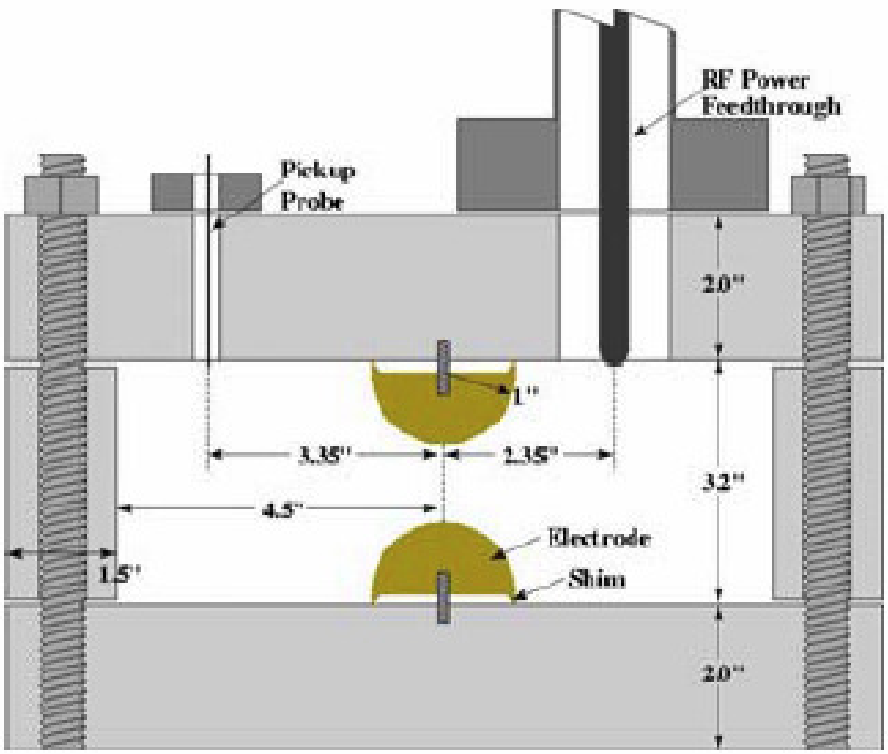
March 9, 2010

Presentation Outline

I. Why High-Pressure RF Cavities?
II. Natural Convection inside RF Cavities
III. Dimensionless Analysis
IV. Temperature and Velocity Results
V. Generalized Dimensionless Results
VI. Why RF Windows?
VII. Design Considerations of RF windows for High-Pressure RF Cavities?
VIII. Conclusions & Recommendations

I. Why High-Pressure RF Cavities?

i) Higher RF gradients and better cavity breakdown behavior are achieved due to the Paschen effect. Increasing gas density reduces the mean free collision path for ions giving them less chance to accelerate to higher energies.



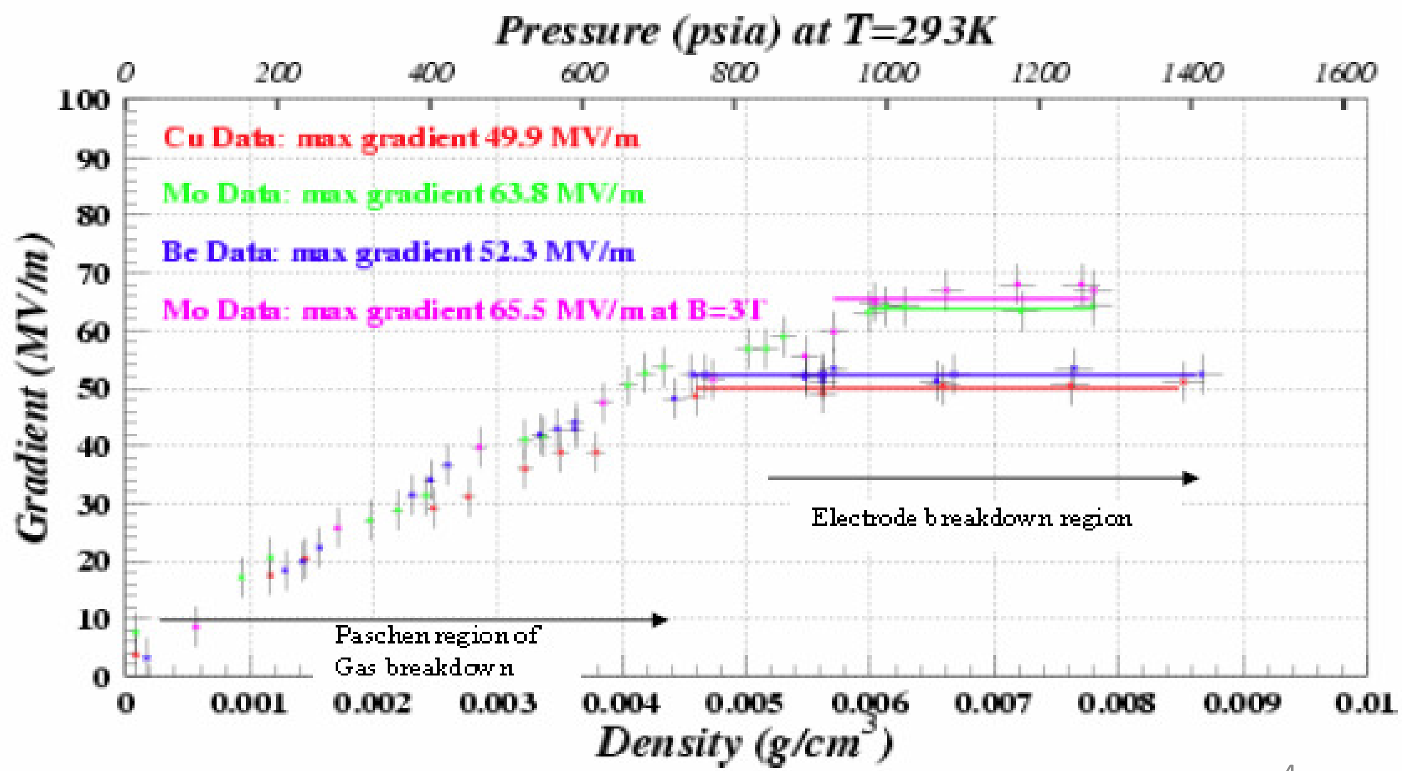
Test cavity with two electrodes

From "Papers and Reports" at www.muonsinc.com

I. Why High-Pressure RF Cavities?

- Unlike vacuum cavities, the gas-pressurized cavity demonstrates no degradation of maximum stable gradient with magnetic fields.
- RF cavity conditioning was much faster in gas cavities compared to vacuum cavities.

From Papers and Reports at www.muonsinc.com



I. Why High-Pressure RF Cavities?

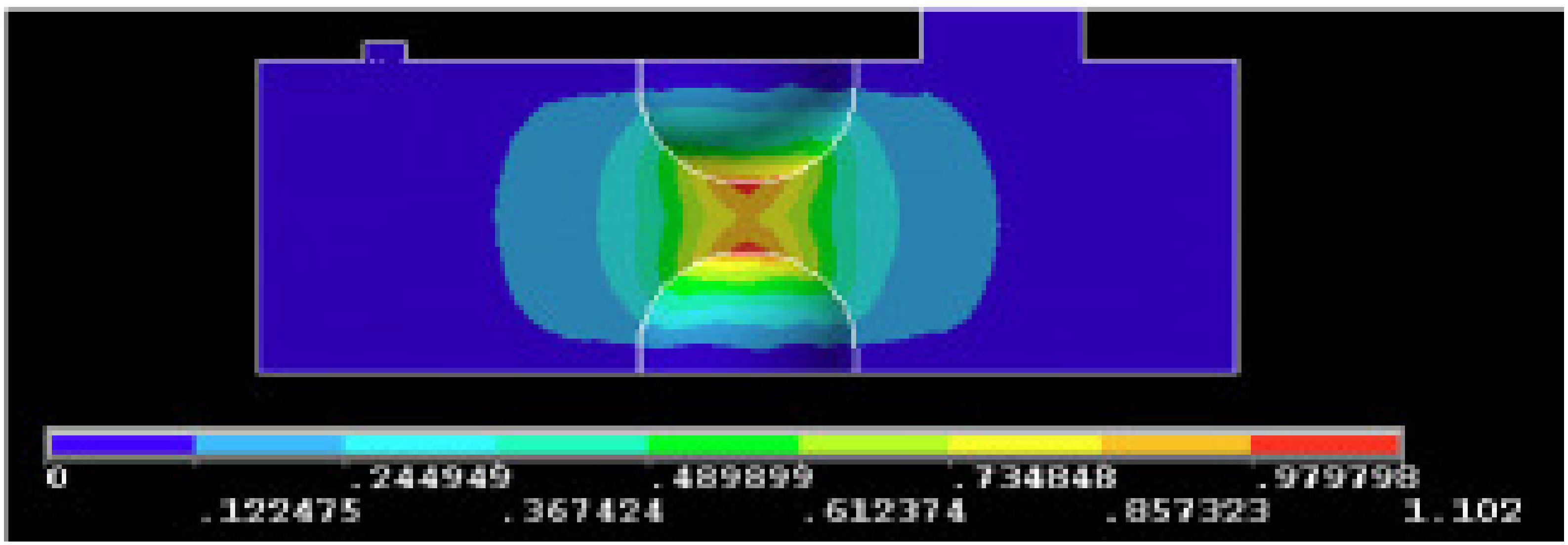
ii) Cooling of the transverse phase-space coordinates of a muon beam can be accomplished by ionization cooling, which requires passing the beam through low-z energy-absorbing material and accelerating structures, both embedded within a focusing magnetic lattice.

- In High-pressure cavities, the energy absorption and energy regeneration happen simultaneously at the cavity. Therefore, the cooling channel length is reduced.

From Papers and Reports at www.muonsinc.com

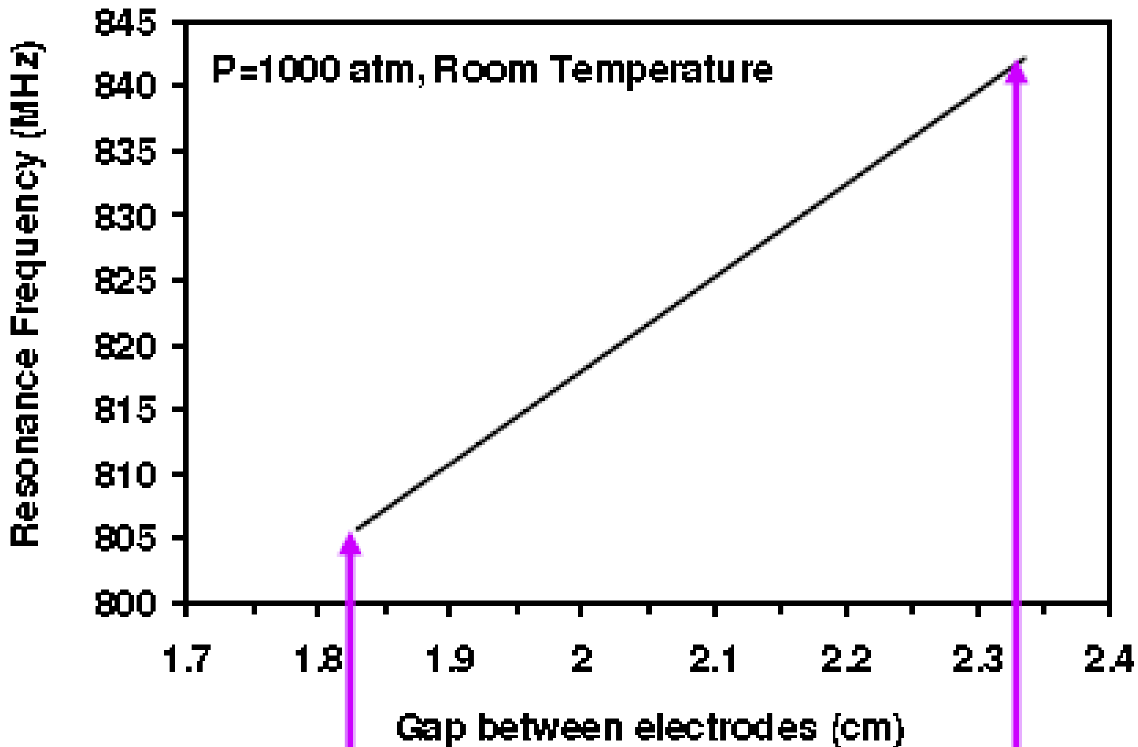
How did we tune it & some frequency results

- One major task was to predict the permittivity of the pressurized hydrogen gas. frequency–pressure curve of an 805 MHz cavity, pressurized with hydrogen gas, was obtained experimentally. Extrapolation determined a relative permittivity value of 1.12 at 77 K and 11 MPa. Modelling showed that the diameter of the cavity, operated at vacuum, should be decreased by 5.5 % to obtain a frequency of 805 MHz when filled with hydrogen gas at 77 K and 11 MPa.



Electric field map in the middle cross section of the test cavity

How did we tune it & some frequency results



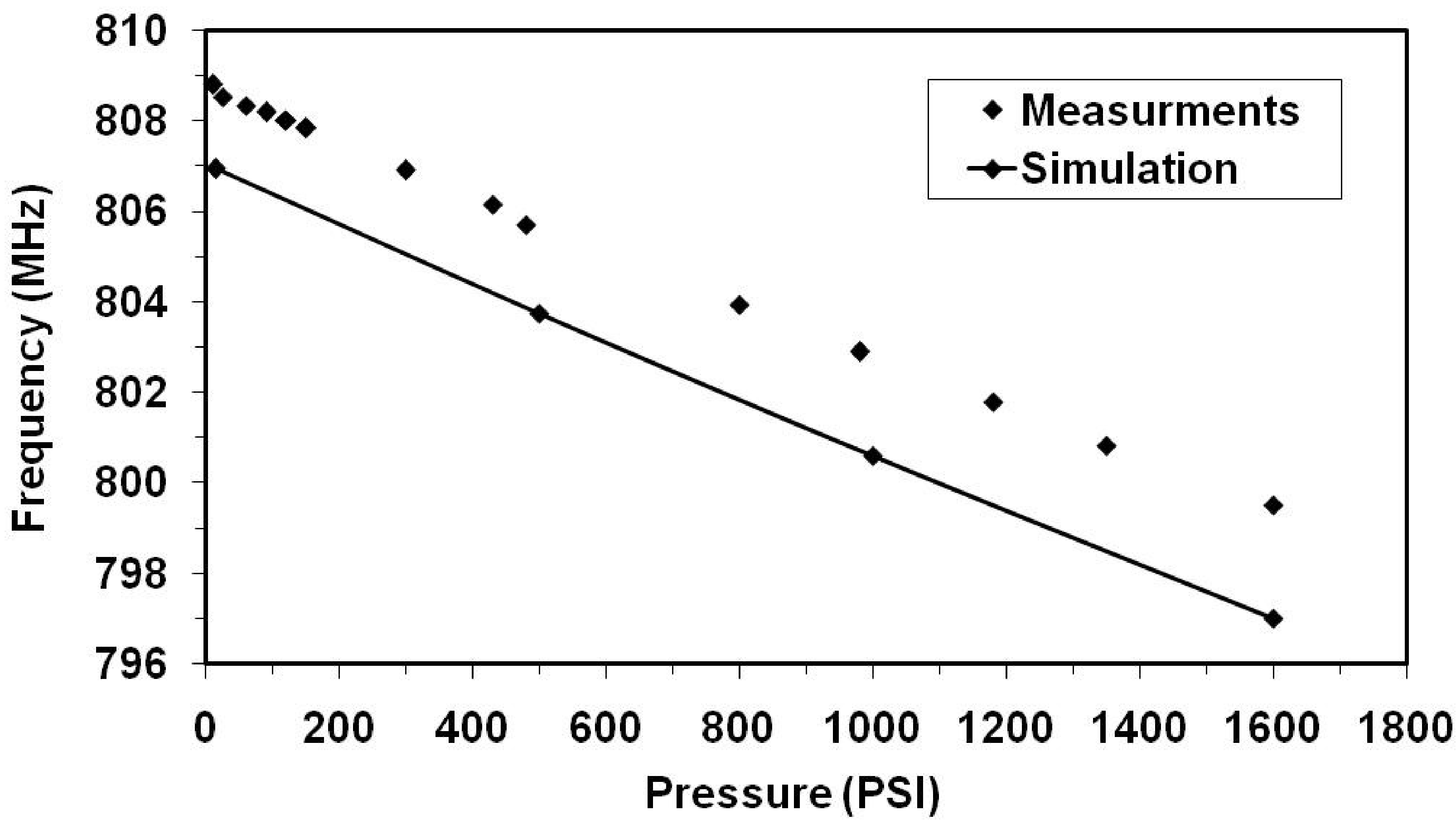
for "Pillbox":

$$\omega_{010} = \frac{2.405}{\sqrt{\mu\varepsilon}R}$$

Tungsten electrodes, each with the 0.1"-thickness copper shim.

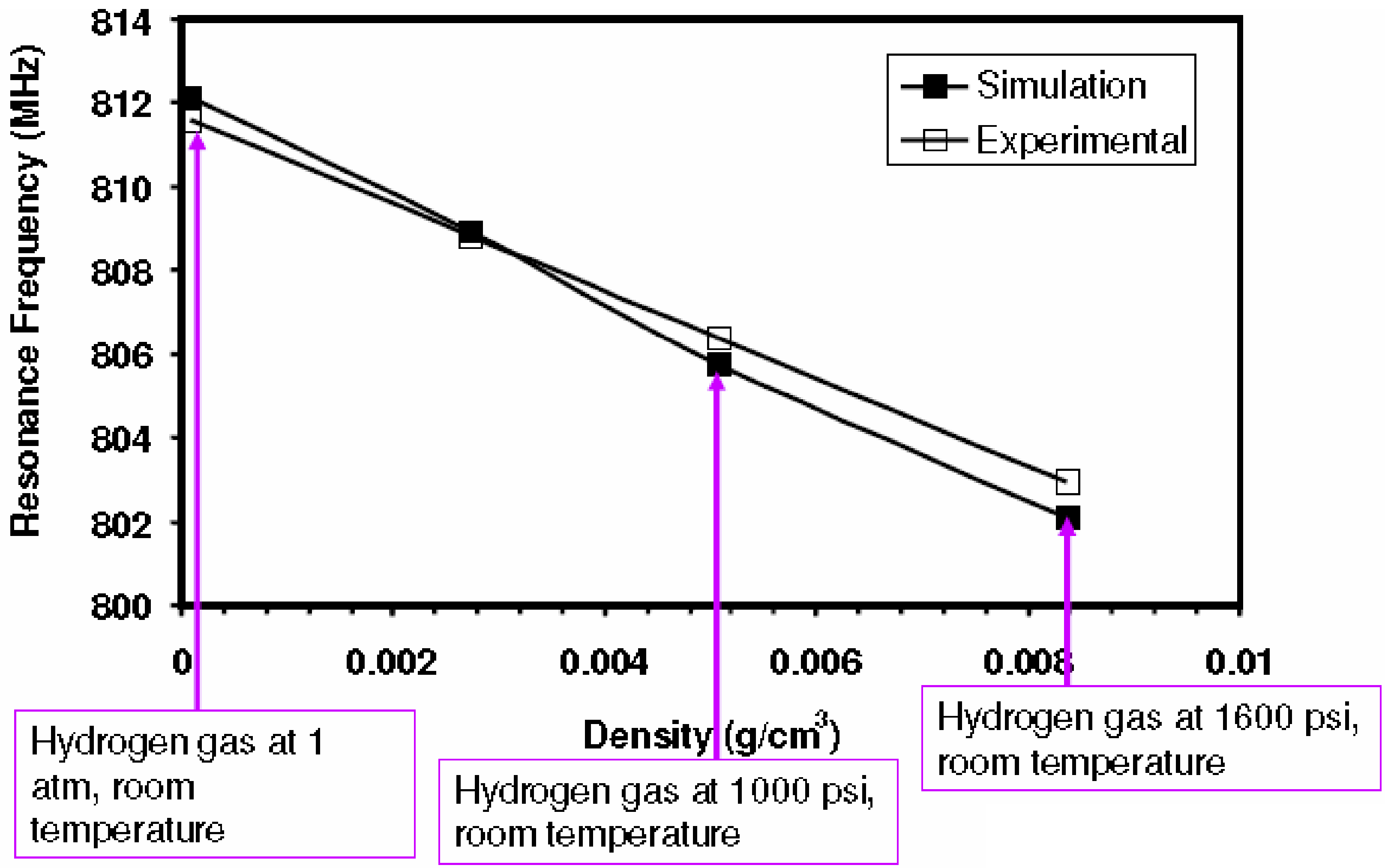
Tungsten electrodes with no shims.

How did we tune it & some frequency results



How did we tune it & some frequency results

Tungsten electrodes, each with a 0.1”-thickness copper shim



II. Natural Convection inside RF Cavities

Governing equations for a flow inside a cylinder subjected to energy deposition (*q*).

Continuity equation:

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

Momentum equation-radial direction:

$$\rho\left[\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u^2{}_\theta}{r}+u_z\frac{\partial u_r}{\partial z}\right]=\frac{-\partial P}{\partial r}+$$

$$\mu\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}-\frac{u_r}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+F_r$$

II. Natural Convection inside RF Cavities

Momentum equation-angular direction:

$$\rho\left[\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+v_z\frac{\partial u_\theta}{\partial z}\right]=\frac{-\partial P}{\partial \theta}+$$

$$\mu\left[\frac{\partial^2 u_\theta}{\partial r^2}+\frac{1}{r}\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r^2}+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]+F_\theta$$

Momentum equation-z direction:

$$\rho\left[\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right]=\frac{-\partial P}{\partial z}+$$

$$\mu\left[\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+F_z$$

II. Natural Convection inside RF Cavities

Heat equation:

$$\rho c \frac{DT}{Dt} = K\nabla^2 T + q + \beta T \frac{DP}{Dt} + \mu\Phi$$

$$\rho c\left[\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z}\right] =$$

$$K\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + q + \beta T \frac{DP}{Dt} + \mu\Phi$$

- **Boundary Conditions:** Velocity at boundary = 0, Temperature at Boundary is fixed at T_o.
- **Initial Conditions:** Initial Velocity = V_o & Initial temperature = T_o

- **To get temperature and velocity solutions: All previous coupled equations should be solved together subjected to the boundary and initial conditions !!!**

III. Dimensionless Analysis

$$r^* = \frac{r}{R} \qquad u^* = \frac{uR}{\alpha} \qquad T^* = \frac{T - T_o}{qD^2 / K}$$

- **Nusselt number (Nu) is the ratio of convection heat transfer to conduction heat transfer across the boundary.**
- **h is the convictive heat transfer coefficient, K is flow thermal conductivity, and D is the diameter of the cavity.**

$$Nu = \frac{hD}{K}$$

u: velocity (m/s)
q: heat generation (w/m^3)
α : thermal diffusivity, α = k / (ρcp), (m^2/s)
v : Kinematic viscosity = μ / ρ, (m^2/s)

III. Dimensionless Analysis

- Rayleigh number (*Ra*) is a parameter associated with the buoyancy driven flow and equals the product of Grashof number (*Gr*) and Prandtl number (*Pr*).

- *Gr* is the approximation of the ratio of buoyancy force to viscous force.

- *Pr* is the ratio of viscous diffusion rate to thermal diffusion rate.

$$Ra = \frac{\beta D^5 \rho c g q}{K^2 v} \qquad Pr = \frac{v}{\alpha}$$

IV. Temperature and Velocity Results

i. Gaseous Hydrogen Cavity

Geometry & Load

Parameter	Value
Diameter of the cavity (*D*)	**29.5 cm**
Radius of the RF window (*R*)	**8.0 cm**
Thickness of the RF window (*t*)	**0.127 mm**
Heat generation per unit length (*q′*)	**178.6 W/m**

- The beam energy was considered as internal heat generation acting throughout a cross sectional circle of 8 cm – radius along the axis of the cavity .

Thermal Properties

Parameter	Value
Gas reference pressure (P)	11.0 MPa
Gas reference temperature (T)	77.0 K
Gas density (ρ)	34.7 kg/m^3
Gas kinematic viscosity (υ)	1.58×10^{-7} m^2/s
Gas specific heat (C_p)	1.55×10^{4} J/kg.K
Gas thermal conductivity (K)	0.098 W/m.K
Gas thermal diffusivity (α)	1.82×10^{-7} m^2/s
Gas coefficient of thermal expansion (β)	0.01299 K^{-1}
Boundary Wall Temperature (BC)	77.0 K
Gas Initial Temperature (IC)	77.0 K

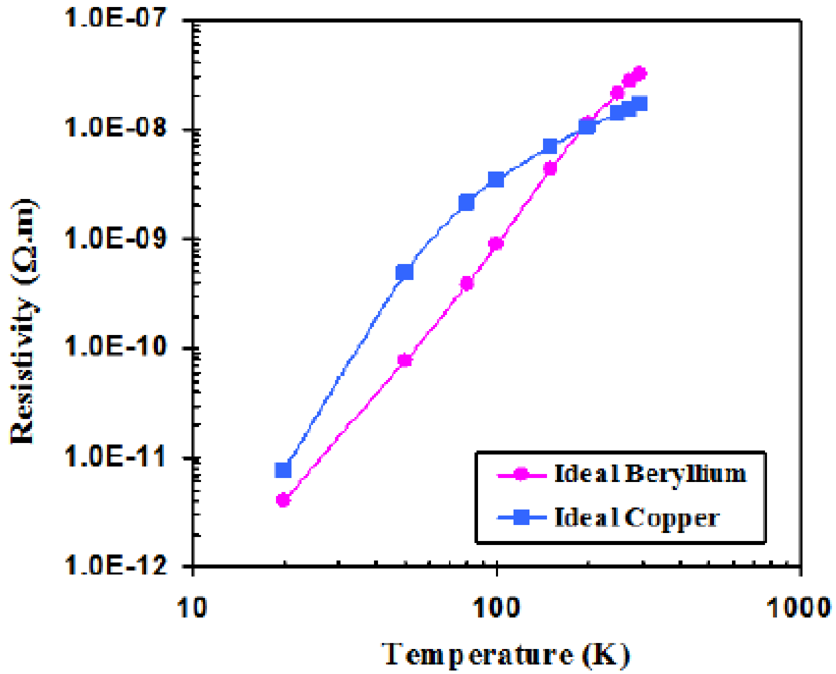
Dimensionless Numbers

Parameter	Value
Prandtl number (*Pr*)	0.87
Rayleigh number (*Ra*)	2.0×10^{14}
Average Nusselt number (*Nu*)	218.0
Heat transfer coefficient (*h*)	79.2 $W/m^2.k$

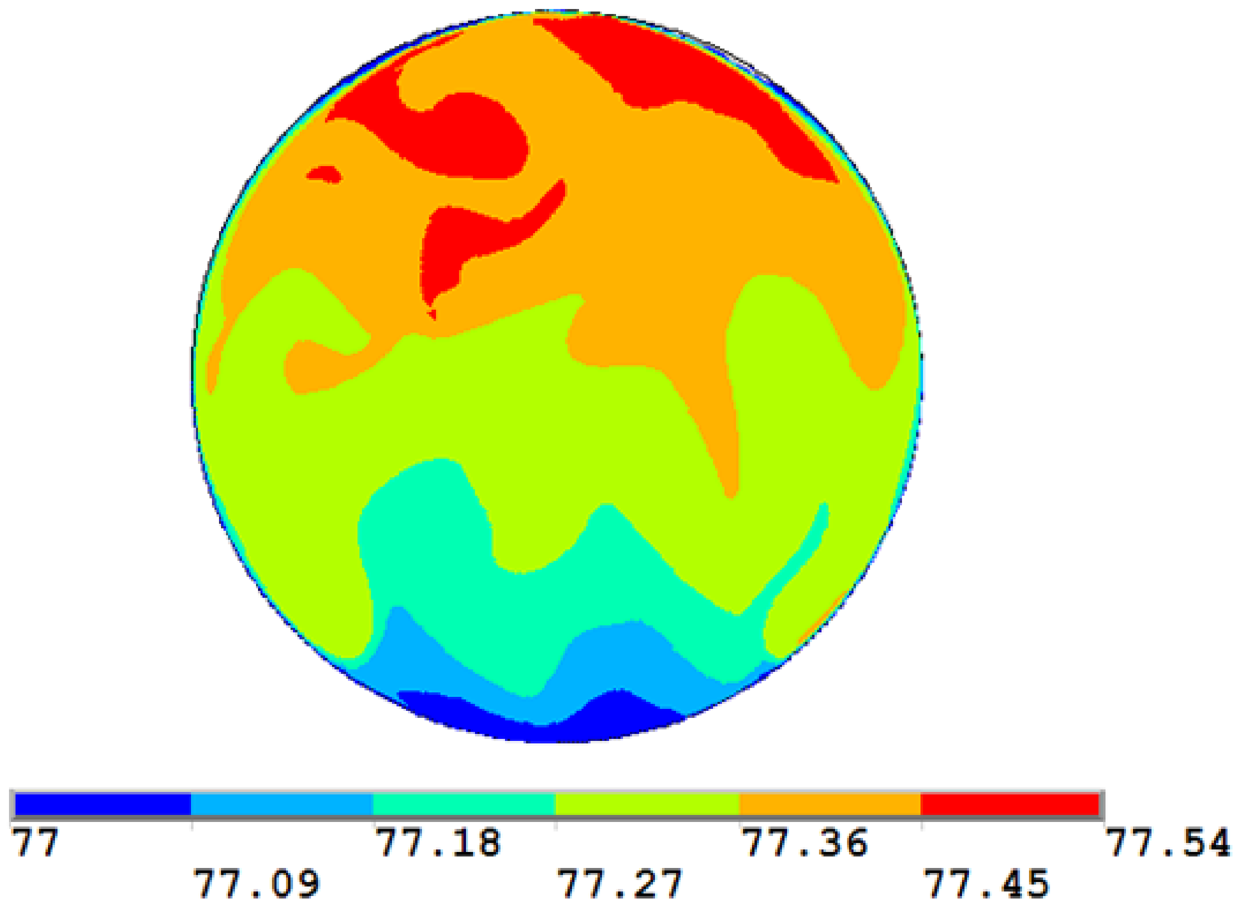
- Heat transfer coefficient (h) and temperature results will be used for thermal designs of RF Cavity windows.

Cavity Parameters

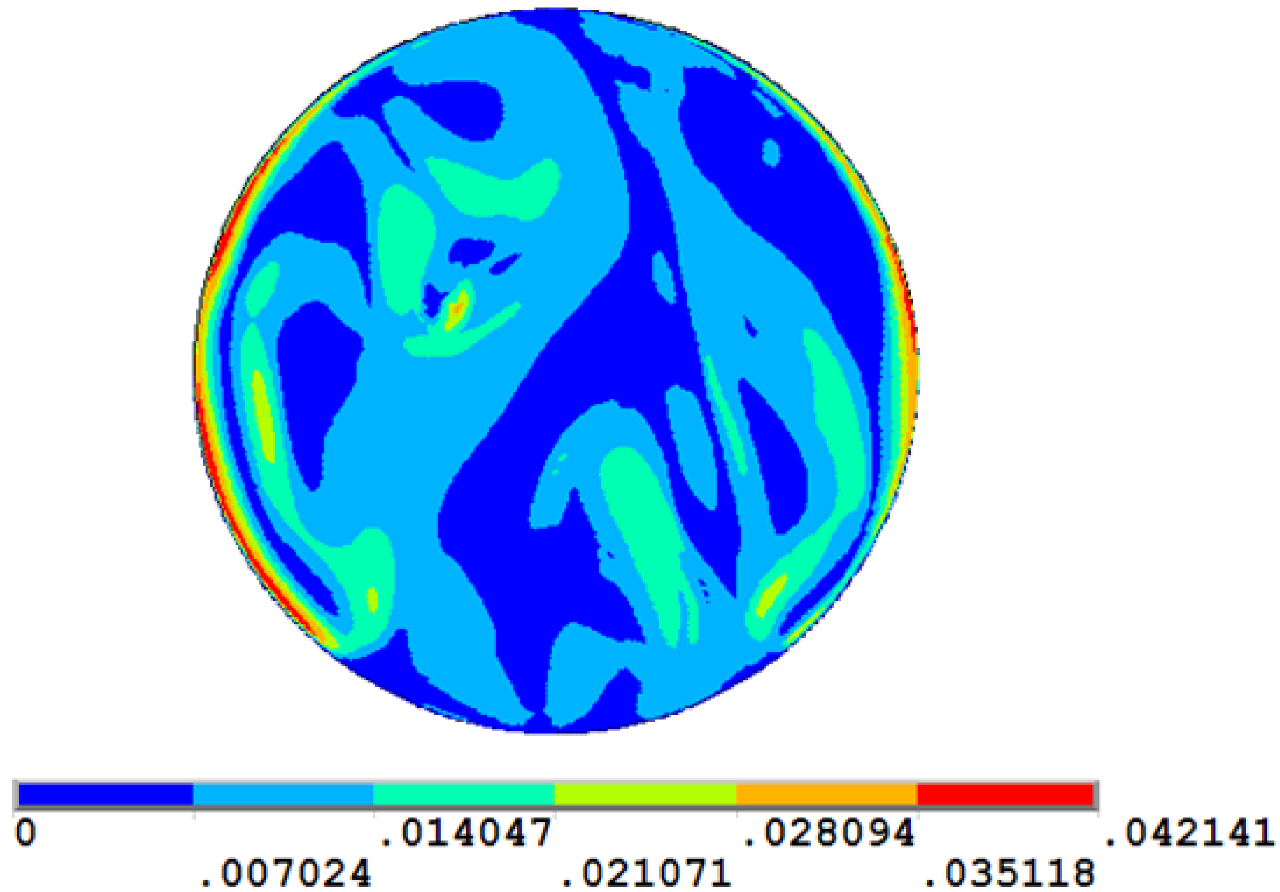
Parameter	Value
On-axis electric field (E_0)	30.0 MV/m
Frequency (f)	805.0 MHz
Quality factor (Q)	5.6×10^4
Electrical conductivity-beryllium	2.56×10^9 (ohm.m)$^{-1}$
Electrical conductivity-copper	4.61×10^8 (ohm.m)$^{-1}$



- Operation of the windows at low temperature values is very attractive.



Contour plot of the flow temperature (K).



Contour plot of the magnitude of the flow velocity (m/s).

IV. Temperature and Velocity Results

ii. Liquid Hydrogen Channel

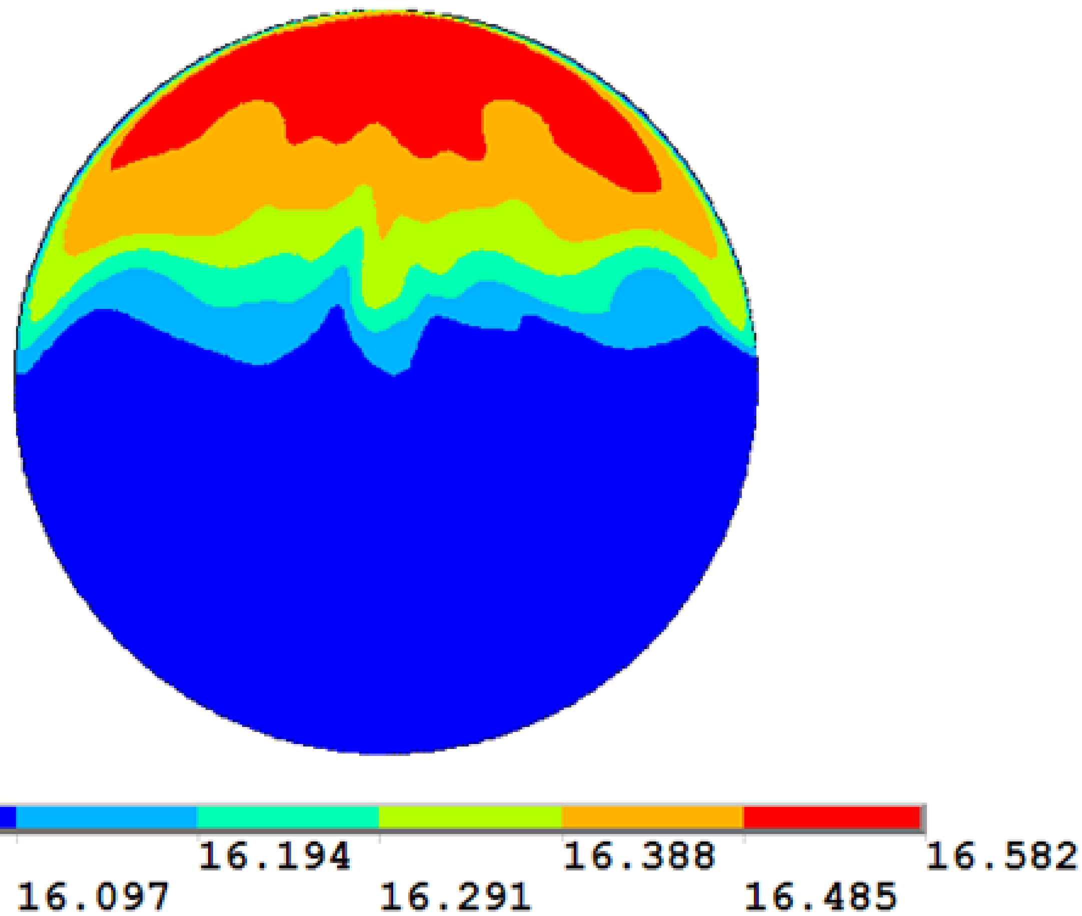
Geometry & Load

Parameter	Value
Diameter of the channel (*D*)	**64.0 cm**
Heat generation per unit length (q')	**3394.4 W/m**

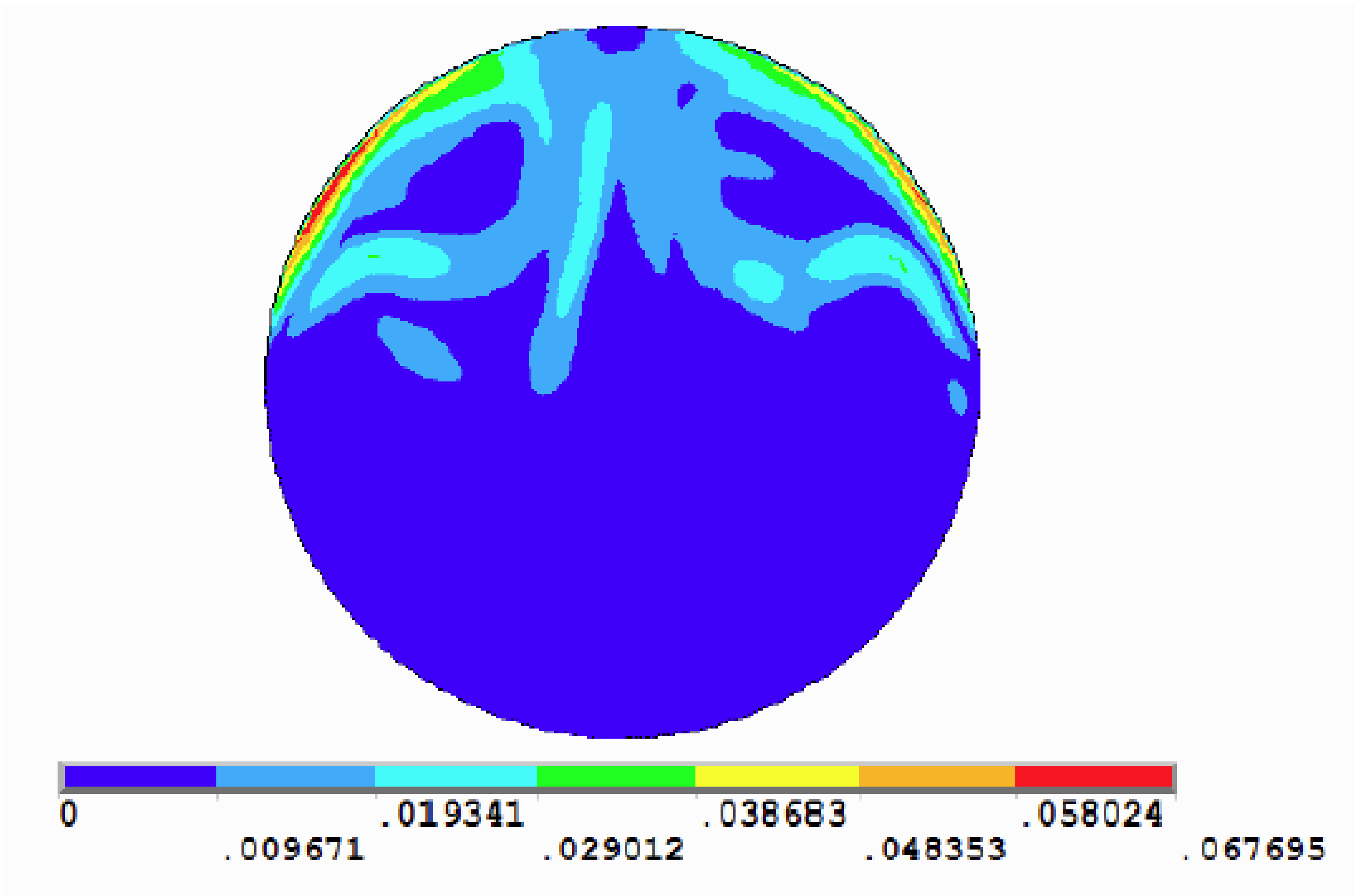
The beam energy was considered as internal heat generation acting throughout a cross sectional circle of 10 cm – radius along the axis of the cavity .

Thermal Properties and Dimensionless Numbers

Parameter	Value
Pressure (*P*)	0.15 MPa
Reference temperature (*T*)	16.0 K
Density (ρ)	75.2472 kg/m^3
Kinematic viscosity (υ)	$2.61\text{x}10^{-7}$ m^2/s
Specific heat (C_p)	$7.42\text{x}10^{3}$ J/kg.K
Thermal conductivity (*K*)	0.0891 W/m.K
Thermal diffusivity (α)	$1.6\text{x}10^{-7}$ m^2/s
Coefficient of thermal expansion (β)	0.0121 K^{-1}
Boundary Wall Temperature (BC)	16.0 K
Initial Temperature (IC)	16.0 K
Prandtl number (*Pr*)	1.64
Rayleigh number (*Ra*)	$3.73\text{x}10^{16}$

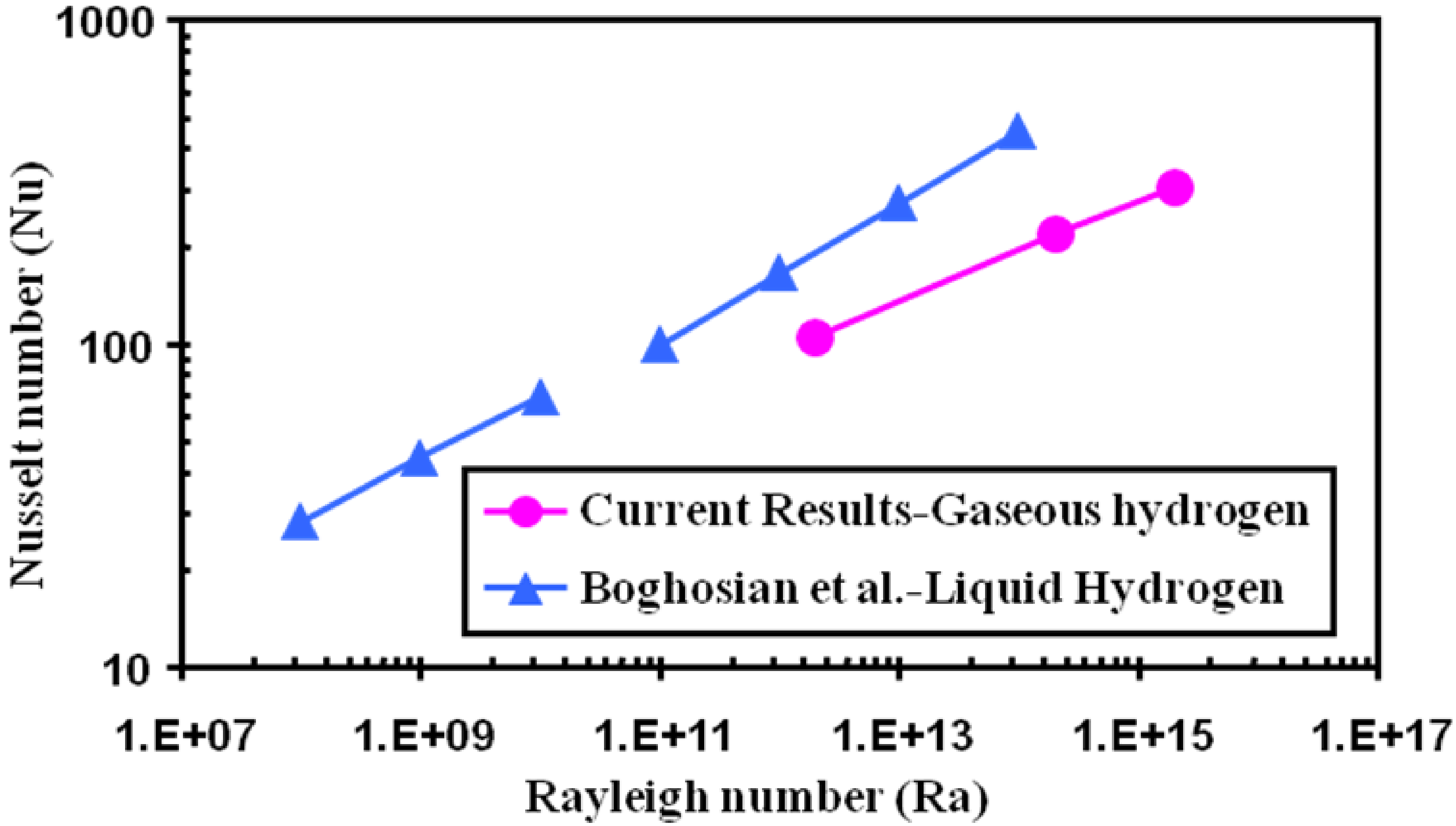


Contour plot of the flow temperature (K).



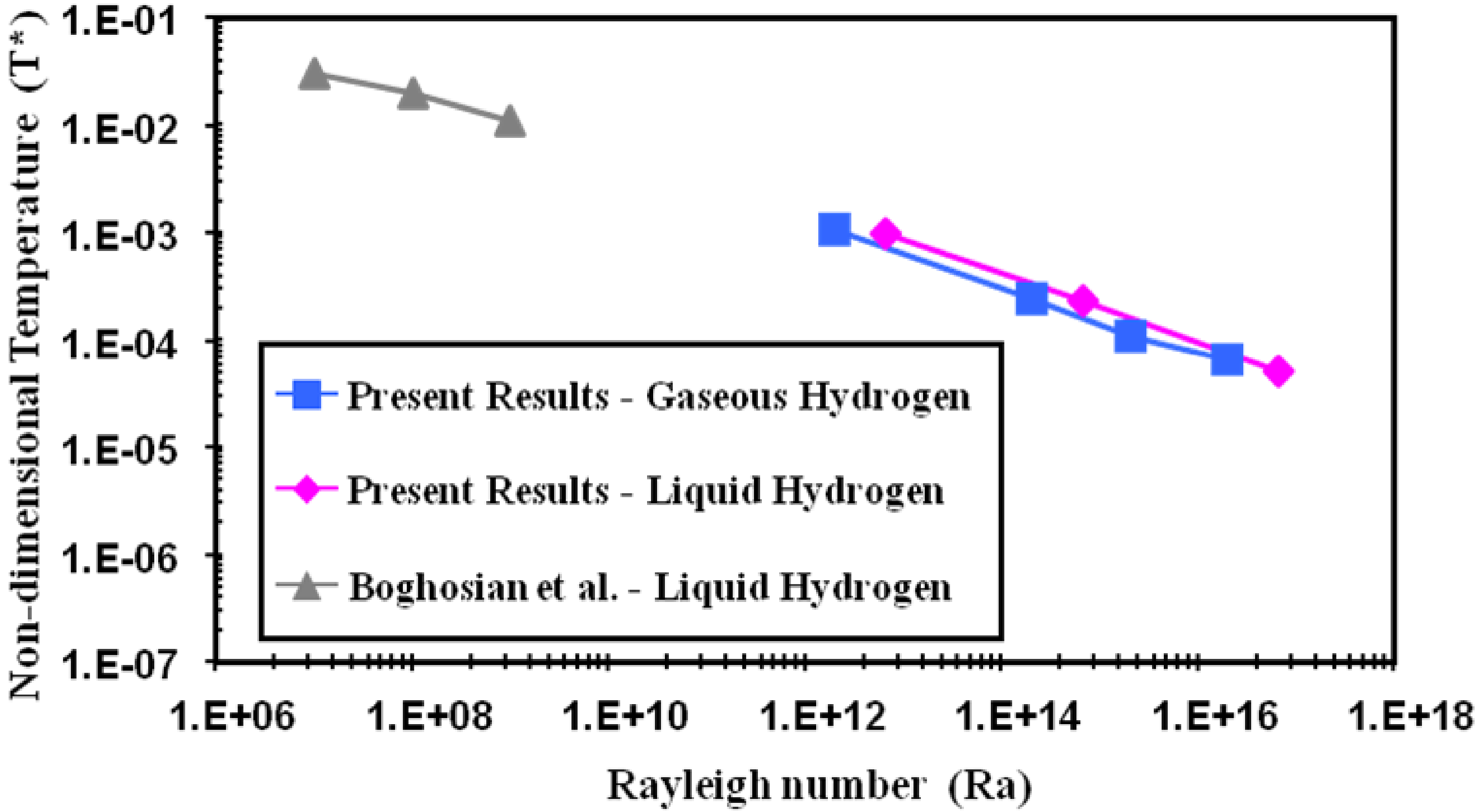
Contour plot of the magnitude of the flow velocity (m/s).

V. Generalized Dimensionless Results



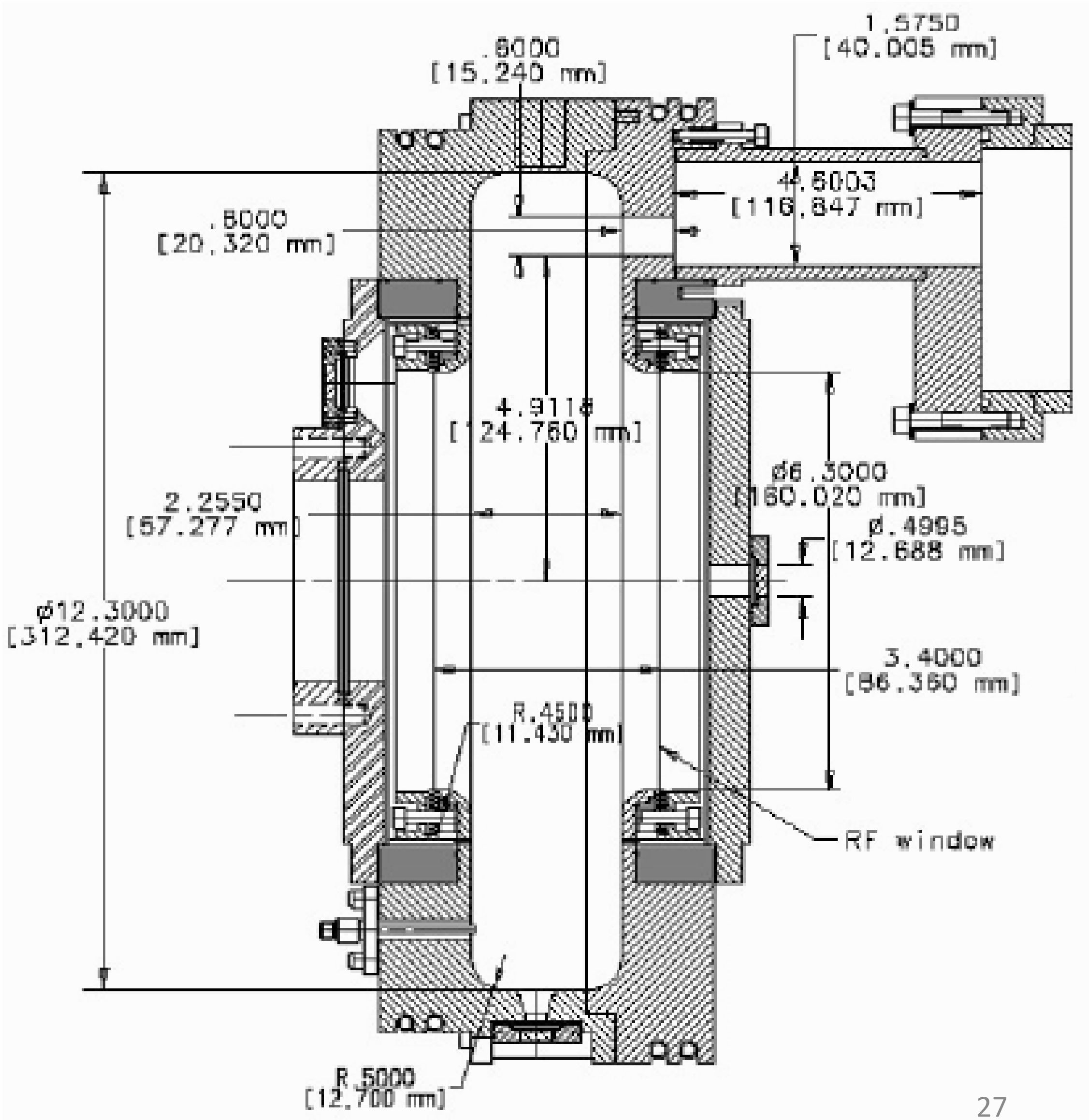
M. Boghosian, "A Numerical Investigation of Laminar and Turbulent Natural Convection in A Cylinder with Heat Generation", M.S. Thesis under supervison of Dr. K. Cassel, MMAE, IIT, July01.

V. Generalized Dimensionless Results



VI. Why RF Windows?

- RF cavity windows are thin metallic windows that close cavity ends to increase the on-axis electric field for a given maximum surface field and to reduce the required RF cavity power.



VI. Why RF Windows?

- Studies of 805 MHz and 201 MHz normal-conducting RF cavities showed that:
  - In open RF cavities, the maximum surface electric field is about double the on-axis electric field.
  - In RF cavities closed by metallic foils, the maximum surface electric field is approximately equal to the on-axis electric field.

- RF cavity windows thus help in increasing the on-axis electric field with respect to a given maximum surface electric field. Dark current is proportional to the surface electric field raised to a high power. Consequently, RF cavity windows allow the operation of cavities at higher gradients.

Design Requirements of RF Windows

- The window should be made of a material of long radiation length to reduce the scattering of the beam as it passes through the window.

- The electrical conductivity of the window material should be relatively high to decrease the power loss (the power dissipated in the window is inversely proportional to the square root of the electrical conductivity of the window material). Decreasing the power loss will decrease the temperature rise in the window, and consequently decrease the out-of-plane deflection and thermal stresses.

- Some aluminum and beryllium alloys are acceptable materials for RF windows.

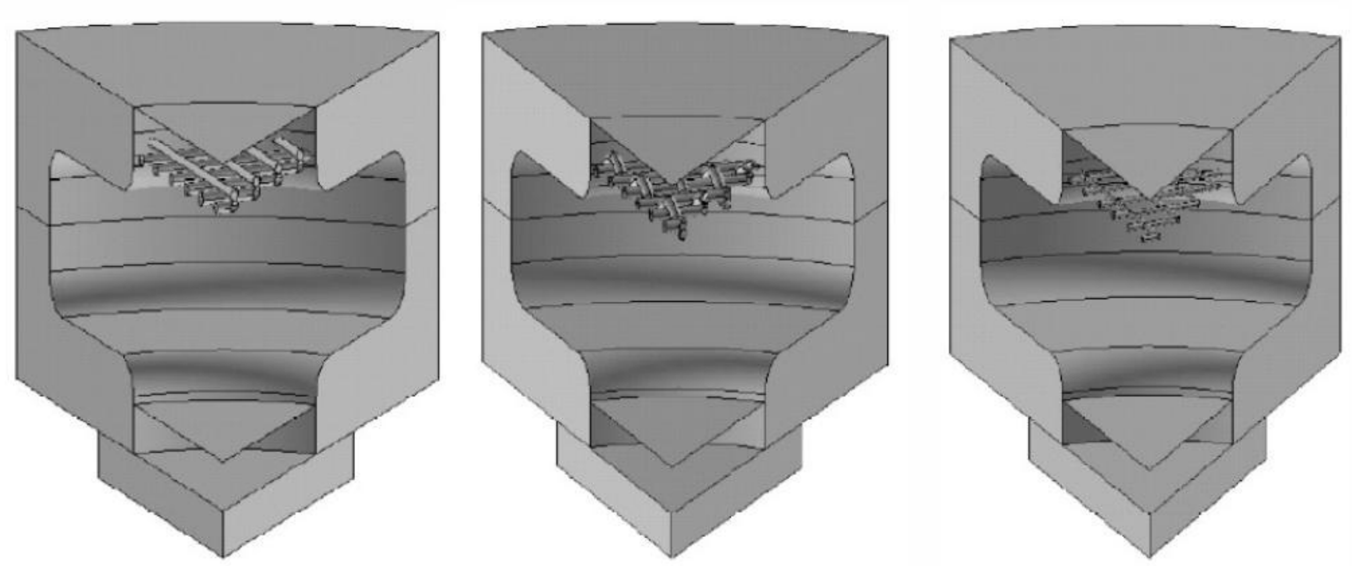
Design Requirements of RF Windows

- Deflection of the 201 MHz cavity window and the corresponding frequency shift are very critical design issues.

- RF heating can cause the windows to deform in the axial direction of the cavity. This out-of-plane deflection must be small enough so that the consequent frequency variation is tolerable. For example, for an 805 MHz cavity, the out-of-plane deflection of the window must be kept below 25 microns to prevent the resonant frequency of the cavity from shifting more than 10 kHz. In the case of 201 MHz cavities, the window diameter is larger than that of the 805 MHz cavity, which implies larger RF-heating-induced deflection.

VII. Design Considerations of RF windows for High-Pressure RF Cavities

- We are proposing to develop RF cavity windows, composed of grids of solid wires, for 201 MHz high-pressure RF cavities. These windows gridded-wire windows are basically arrays of wires arranged and/or overlapped in specific patterns.



Grid 1 Grid 2 Grid 3

RF windows for High-Pressure RF Cavities

- **The generalized dimensionless results of Temperature. vs. Ra and Nu. vs. Ra will aid in the thermal design of the windows.**
- The gridded-wire windows will handle RF heating with negligible displacement due to the structural integrity of the wiring patterns.
- The gas will pass between the cavities through the spaces in the gridded-wire window and there will be no need to control/monitor the pressure of each cavity.
- There will be no thin foils and no need to control/monitor the deflection of any foil.

RF windows for High-Pressure RF Cavities

- Design requirements of windows for a normal conducting 805 MHz vacuum cavity for muon acceleration can be satisfied using a gridded-tube window.

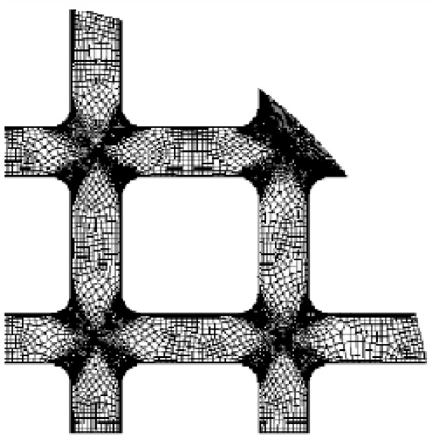
- An example window was composed of an array of four tubes in the x-direction intersecting with another array of four tubes in the y-direction. For this design, with a tube wall thickness of 254 μm, and with helium gas flowing through the tubes, the maximum calculated temperature rise in the grid was only 7.7 ^{o}C, the maximum displacement was only 13 μm and the maximum Von Mises stress was 30.8 MPa, with a yield-stress safety factor of 9. The results showed that gridded-tube windows can be excellent windows for 805 MHz vacuum cavities.

RF windows for High-Pressure RF Cavities

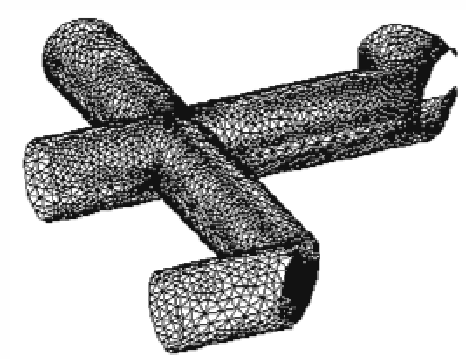
- Iterative testing of the mesh density shows that the structural analysis requires the highest mesh intensity. So the meshing order is opposite to the analysis order. ***[rule of thumb].***

STRUCTURAL ANALYSIS ← **THERMAL CONVECTION ANALYSIS** ← ELECTROMAGNETIC ANALYSIS

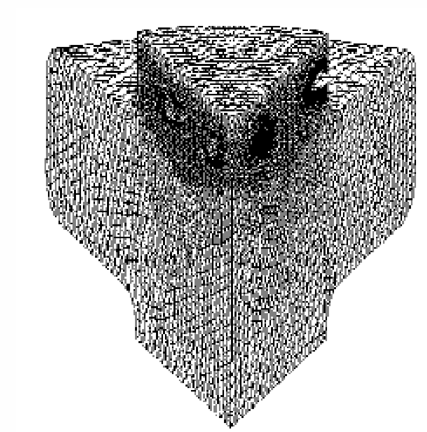
STRUCTURAL MESH → **THERMAL CONVECTION MESH** → ELECTROMAGNETIC MESH



Structural FEA mesh



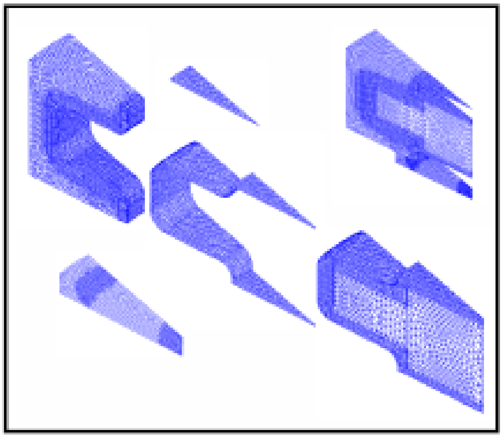
Thermal FEA mesh



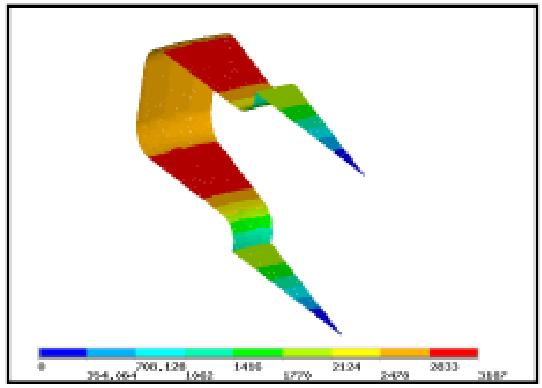
Electromagnetic FEA mesh

RF windows for High-Pressure RF Cavities

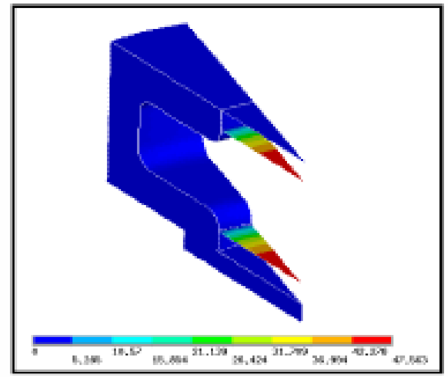
Average power loss in the cavity=500 W. Be-window thickness=127μm, tube DIA=0.9525 cm, Tube wall thickness= 254 μm



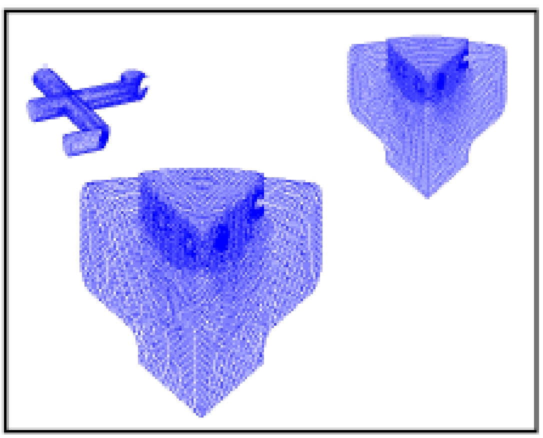
FEA mesh



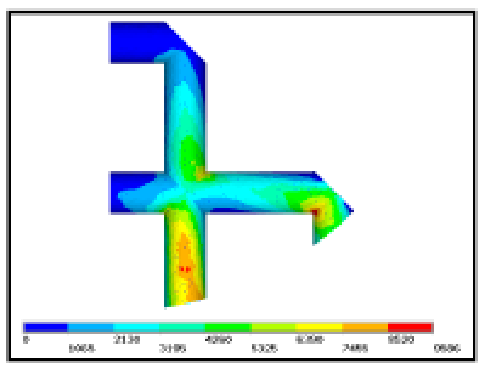
Heat flux distribution (W/m^2)



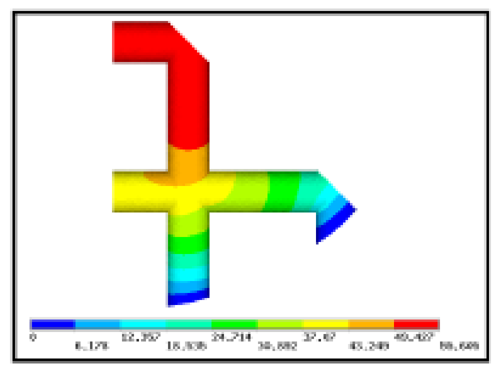
Temperature rise (^{0}C)



FEA mesh

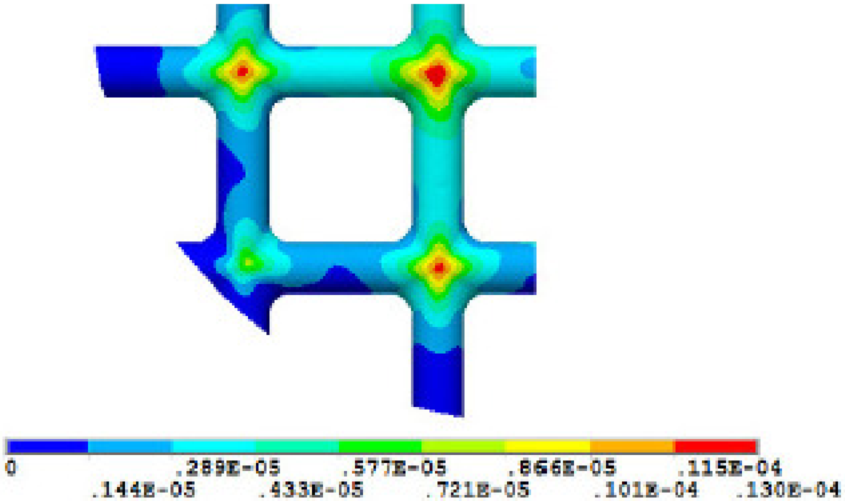


Heat flux distribution (W/m^2)

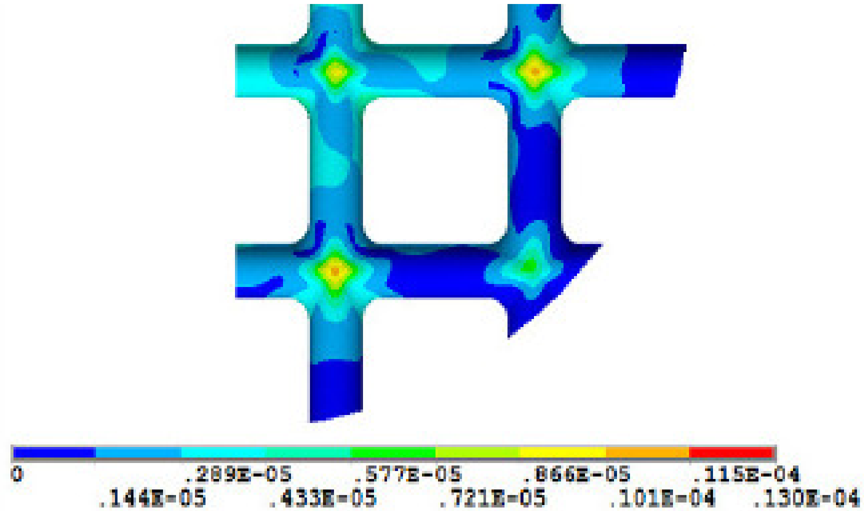


Temperature rise (^{0}C)

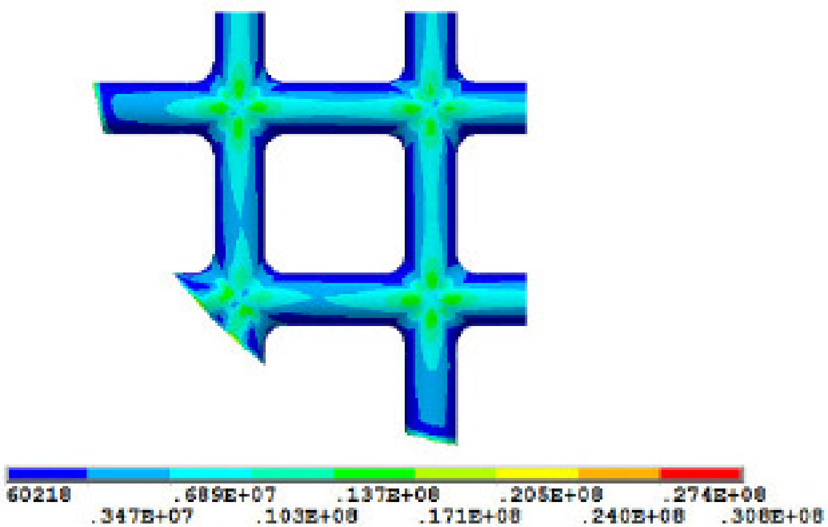
RF windows for High-Pressure RF Cavities



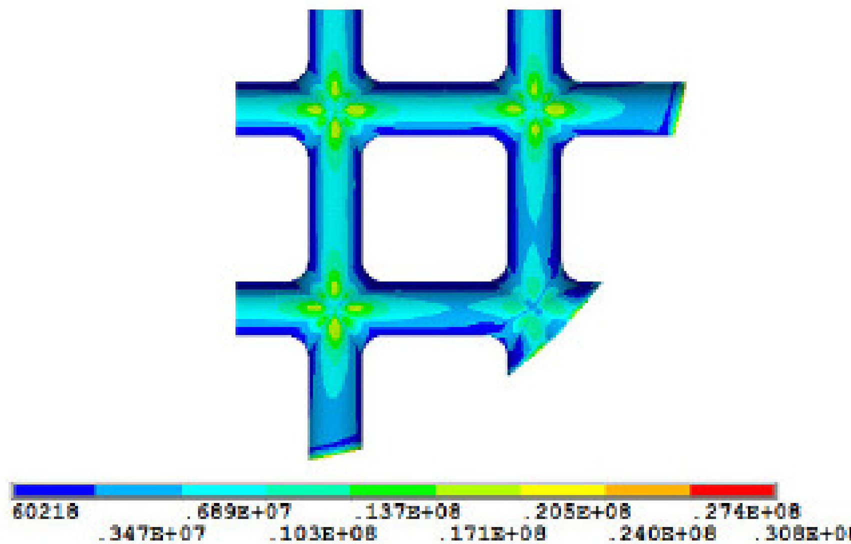
Displacement (m), external side



Displacement (m), internal side



VonMises stress (Pa), external side



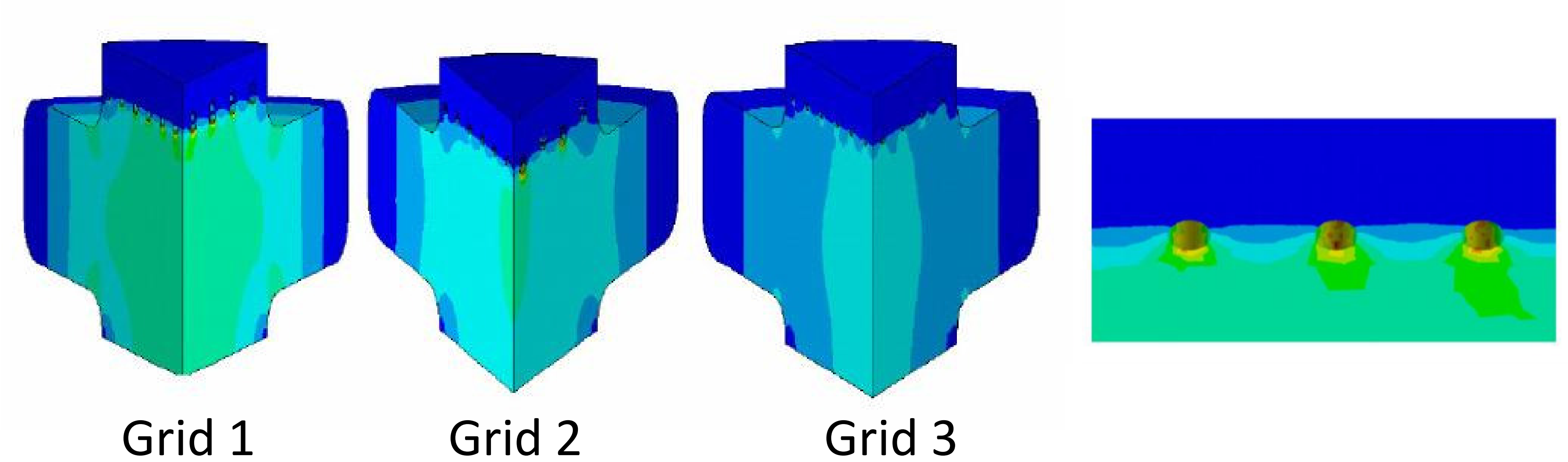
VonMises stress (Pa), internal side

RF windows for High-Pressure RF Cavities

- The design of the gridded window should terminate the electric field or cause negligible electric field leakage into the region beyond the window. This allows adjacent RF cavities to operate independently of each other.

- It was found that various grid configurations were able to minimize field leakage. Also, for many gridded-tube configurations, the electric field distribution was smooth in the cavity with little distortion in the vicinity of the grid. Gridded-tube windows were found to be feasible for use as RF cavity windows provided that tube geometry and grid configuration were chosen appropriately .

RF windows for High-Pressure RF Cavities



- The surface electric field enhancements are 2.2 for grid 1, 2.1 for grid 2, and 2.9 for grid 3. There are many ways to decrease the surface field enhancement including adding more wires, increasing the wire diameter in the grid, or other configurations.

- Resonant frequencies are 805.2 MHz for grid 1, 803.0 MHz for grid 2, and 808.1 MHz for grid 3. Quality factor values are 17,620 for grid 1, 17,492 for grid 2, and 17,921 for grid 3.

RF windows for High-Pressure RF Cavities

- The plans for muon cooling are currently gaining significant interest from Fermilab . Realistic plans require feasible production of various engineered components. The ionization cooling channel for muon colliders, and neutrino factories, and stopping muon beam experiments requires many hundreds of RF cavity windows.

- The current RF cavity designs are “pill box” RF cavities closed by very thin beryllium foils. Thin beryllium windows are very expensive. They now cost $25,000 each. The gridded-wire windows will cost a very small fraction of that. Also, RF heating from the cavities causes these foils to deform out-of-plane and consequently detune the cavity, which in turn, will degrade the operation of the cavity. The gridded-wire designs can solve this problem . They are excellent technical and economical alternatives.

Conclusions & Recommendations

- **Generalized dimensionless parameters of the natural convection, driven by energy deposition inside high-pressure RF cavities, have been obtained.**
- **The solutions of both the temperature (Temperature.vs.Ra) and convective heat transfer coefficient (Nu.vs.Ra) are useful for the thermal design of high-pressure RF cavity windows.**
- **The heating and deformation of RF cavity windows are critical design issues. Structural, fabrication, and testing studies of the windows are required.**

Questions/Discussion

I hope you enjoyed my presentation.

- Thanks to Muons, Inc. Staff.
- Thanks to Al Moretti and Milorad Popovic.
- Thanks to MuCool Collaboration.